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A universal seesaw mechanism in five dimensions

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Abstract

We show the universal seesaw in the extra dimension setup, where three extra vector-like fields exist in the 5D bulk with heavy masses. We take the framework of the left-right symmetric model. The universal seesaw formula is easily obtained as a replacement of the vector-like mass in 4D case M_i to $2M_* \tan[\pi R M_i]$ (M_* : 5D Planck scale, M_i : vector-like bulk mass, and R : compactification radius). The smallness of Dirac neutrino mass can be naturally explained in the 5D setup. We also show the Majorana neutrino case.

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1 Introduction

The origin of quark and lepton masses is still a mystery, which can be a very important key for searching a fundamental theory beyond the standard model (SM). Why are fermion masses except for top quark mass much smaller than electroweak breaking scale, and why are neutrino masses much smaller than other fermion masses? The ordinal seesaw mechanism was suggested to explain the smallness of neutrino masses[1]. The smallness of quark and lepton masses (except for top quark mass) comparing to the electroweak breaking scale can be explained in a different framework, where vector-like extra fermions are introduced with heavy masses. This mechanism is so-called universal seesaw mechanism[2]-[8]. The extra fermions have vector-like masses so that they have no effects to the electroweak precision measurements. The left-right symmetric model with a gauge group, $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, is one example which can have a universal seesaw structure. For the top quark mass which is the same order as electroweak symmetry breaking, we should take special setup[7][†].

In this paper we show the universal seesaw mechanism in the flat extra dimension setup[9]. We show the 5D universal seesaw in the framework of the left-right symmetric model. We introduce extra vector-like ($SU(2)_L \times SU(2)_R$ singlet) fields in the 5D bulk, and other fermions on the 4D brane. The 5D universal seesaw formula is easily obtained as a replacement of the vector-like mass in 4D case M_i to $2M_* \tan[\pi R M_i]$ (M_* : 5D Planck scale, M_i : vector-like bulk mass, and R : compactification radius)[10]. As for the neutrinos, the smallness of Dirac neutrino masses can be naturally explained in the 5D setup. Only Dirac neutrino masses can be very small when the bulk neutrino masses are close to a half of compactification scale. We will also show the Majorana neutrino case, where 5D seesaw mechanism works. For the heavy top quark mass, the vector-like extra “top” quarks should be localized on the 4D brane.

This paper is organized as follows. In section 2, we show a brief review of 4D universal seesaw. Section 3 shows the 5D universal seesaw formula. For the neutrino sector, we will study both Dirac and Majorana neutrino cases. Section 4 shows the summary.

2 4D universal seesaw mechanism

At first we briefly review the 4D universal seesaw mechanism in the left-right symmetric model, which has a gauge group, $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The quark sector is given by

$$Q_{Li} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L = (3, 2, 1, 1/3), \quad Q_{Ri} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R = (3, 1, 2, 1/3), \quad (1)$$

[†]Another approach for the large top mass has been studied in S_3 flavor symmetry[5].

$$U_{Li} = (3, 1, 1, 4/3), \quad U_{Ri} = (3, 1, 1, 4/3), \quad (2)$$

$$D_{Li} = (3, 1, 1, -2/3), \quad D_{Ri} = (3, 1, 1, -2/3). \quad (3)$$

The Higgs sector are

$$H_L = (1, 2, 1, 1), \quad H_R = (1, 1, 2, 1), \quad (4)$$

which take vacuum expectation values (VEVs) as

$$\langle H_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix} = (0, v_L, 0, 0), \quad \langle H_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix} = (0, 0, v_R, 0), \quad (5)$$

where $v_R \gg v_L$ must be satisfied with $v_R \geq \mathcal{O}(10^4)$ GeV from the experimental search of W_R [11].

The most general Yukawa interactions in the quark sector are

$$\mathcal{L}_{mass}^q = y_{ij}^{u,L} \overline{Q}_{Li} \tilde{H}_L U_{Rj} + y_{ij}^{u,R} \overline{Q}_{Ri} \tilde{H}_R U_{Lj} + M_{\mathcal{U}ij} \overline{U}_{Li} U_{Rj} + \text{h.c.} \quad (6)$$

$$+ y_{ij}^{d,L} \overline{Q}_{Li} H_L D_{Rj} + y_{ij}^{d,R} \overline{Q}_{Ri} H_R D_{Lj} + M_{\mathcal{D}ij} \overline{D}_{Li} D_{Rj} + \text{h.c.}, \quad (7)$$

where $\tilde{H} = \epsilon H^*$ and M 's are bare mass parameters. Without loss of generality, $M_{\mathcal{U}ij}$ ($M_{\mathcal{D}ij}$) can be a real diagonal matrix, $\text{diag}(M_U, M_C, M_T)$ ($\text{diag}(M_D, M_S, M_B)$), through a bi-unitary transformation.

Then, the up- and down-type mass matrices are given by

$$(\bar{u}, \bar{U})_L \begin{pmatrix} 0 & y^{u,L} v_L^* \\ y^{u,R\dagger} v_R & M_{\mathcal{U}} \end{pmatrix} \begin{pmatrix} u \\ U \end{pmatrix}_R, \quad (\bar{d}, \bar{D})_L \begin{pmatrix} 0 & y^{d,L} v_L \\ y^{d,R\dagger} v_R^* & M_{\mathcal{D}} \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix}_R, \quad (8)$$

respectively. We take real VEVs, for simplicity, and note $m_{Lij}^u = y_{ij}^{u,L} v_L$, $m_{Rij}^u = y_{ij}^{u,R} v_R$, $m_{Lij}^d = y_{ij}^{d,L} v_L$, and $m_{Rij}^d = y_{ij}^{d,R} v_R$. Since we have already used the degrees of freedom for $M_{\mathcal{U},\mathcal{D}}$ to be a flavor diagonal, field redefinitions remain only in $Q_{L,R}$. These field redefinitions can make $m_{L,R}$ triangular matrices as

$$m_{L,R} = \begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}, \quad (9)$$

where \times stands for non-zero element[7].

The $|m_L^{u,d}|$ is of order 100 GeV and $|m_R^{u,d}| \geq 10^4$ GeV. When $|M_{\mathcal{U},\mathcal{D}}| \gg |m_R^{u,d}|$, light three generation quarks' masses are given by

$$(M_{light}^u)_{ij} \simeq -(m_L^u)_{ik} \frac{1}{M_{\mathcal{U}k}} (m_R^{uT})_{kj}, \quad (M_{light}^d)_{ik} \simeq -(m_L^d)_{ik} \frac{1}{M_{\mathcal{D}k}} (m_R^{dT})_{kj}, \quad (10)$$

which is so-called universal seesaw formula. We assume m_L^u and m_R^u are diagonal matrices of the flavor base from now on. We do not make any particular assumption

on the structure of the down-type Yukawa matrix, which is the origin of flavor mixings in the quark sector.

Considering the bottom quark mass, the scale of $|M_B|$ is expected to be of order $10^2 \times |m_R^b|$. When $|M_B| \sim |M_S| \sim |M_D| \sim |M_T| \sim |M_C| \sim |M_U|$, other quark masses can be reproduced through the universal seesaw with the mass hierarchies of $m_{L,R}^\dagger$ except for the top quark mass. In order to obtain the suitable magnitude of top quark mass, $|m_R^t|$ should be the same order as (or even larger than) $|M_T|$ [7]. In this case, the top quark and heavy extra T, \bar{T} quark masses are given by

$$m_t \simeq \frac{m_L^t m_R^t}{\sqrt{m_R^2 + M_T^2}}, \quad m_T \simeq \sqrt{m_R^2 + M_T^2}, \quad (11)$$

respectively. This suggests $m_t = \mathcal{O}(10^2)$ GeV.

Next, let us show the lepton sector, which is give by

$$L_{Li} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_R = (1, 2, 1, -1), \quad L_{Ri} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_R = (1, 1, 2, -1), \quad (12)$$

$$N_{Li} = (1, 1, 1, 0), \quad N_{Ri} = (1, 1, 1, 0), \quad (13)$$

$$E_{Li} = (1, 1, 1, -2), \quad E_{Ri} = (1, 1, 1, -2). \quad (14)$$

The most general Yukawa interactions in the lepton sector are given by

$$\begin{aligned} \mathcal{L}_{mass}^l &= \mathcal{L}_0 + \mathcal{L}' \\ \mathcal{L}_0 &= y_{ij}^{e,L} \overline{L}_{Li} H_L E_{Rj} + y_{ij}^{e,R} \overline{L}_{Ri} H_R E_{Lj} + M_{\mathcal{E}ij} \overline{E}_{Li} E_{Rj} + \text{h.c.} \\ &+ y_{ij}^{\nu,L} \overline{L}_{Li} \tilde{H}_L N_{Rj} + y_{ij}^{\nu,R} \overline{L}_{Ri} \tilde{H}_R N_{Lj} + M_{Nij} \overline{N}_{Li} N_{Rj} + \text{h.c.} \end{aligned} \quad (15)$$

$$\begin{aligned} \mathcal{L}' &= \hat{y}_{ij}^{\nu,L} \overline{L}_{Li} \tilde{H}_L N_{Lj}^c + \hat{y}_{ij}^{\nu,R} \overline{L}_{Ri} \tilde{H}_R N_{Rj}^c \\ &+ M_{NLij} \overline{N}_{Li}^c N_{Lj} + M_{NRij} \overline{N}_{Ri}^c N_{Rj} + \text{h.c.} \end{aligned} \quad (16)$$

\mathcal{L}_0 conserves the lepton number while \mathcal{L}' breaks the lepton number.

When $\mathcal{L}' = 0$, the neutrino and charged lepton mass matrices are given by

$$(\bar{\nu}, \bar{N})_L \begin{pmatrix} 0 & y^{\nu,L} v_L^* \\ y^{\nu,R\dagger} v_R & M_N \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}_R, \quad (\bar{e}, \bar{E})_L \begin{pmatrix} 0 & y^{e,L} v_L \\ y^{e,R\dagger} v_R^* & M_{\mathcal{E}} \end{pmatrix} \begin{pmatrix} e \\ E \end{pmatrix}_R, \quad (17)$$

which are similar to the quark mass matrices. We take real VEVs and note $m_{L,R}^\nu = y^{\nu,L,R} v_{L,R}$, $m_{L,R}^e = y^{e,L,R} v_{L,R}$. We can expect $|m_L^{\nu,e}| = \mathcal{O}(10^2)$ GeV and $|m_R^{\nu,e}| \geq \mathcal{O}(10^4)$ GeV. The masses of light three generation leptons are given by

$$(M_{light}^\nu)_{ij} \simeq -(m_L^\nu)_{ik} \frac{1}{M_{Nk}} (m_R^{\nu T})_{kj}, \quad (M_{light}^e)_{ij} \simeq -(m_L^e)_{ik} \frac{1}{M_{\mathcal{E}k}} (m_R^{e T})_{kj}, \quad (18)$$

[†]It is possible to reproduce quark and lepton mass hierarchies by the mass hierarchies of $M_{\mathcal{U},\mathcal{D}}$, but in this paper we take a standing point that vector-like masses are all the same order.

when $|m_R^{\nu,e}| \ll |M_{\mathcal{N},\varepsilon}|$. Similar to the down-type quark sector, tau mass suggests $|M_\tau| \simeq 10^2 \times |m_R^\tau|$. The mass hierarchy of $m_{L,R}^e$ can realize the mass hierarchy of the three generation lepton masses when $|M_E| \sim |M_\mu| \sim |M_\tau|$. For the suitable tiny Dirac neutrino masses, $|M_{\mathcal{N}}|$ must be extremely larger than other vector-like masses as $|M_{\mathcal{N}}| \simeq 10^{12} \times |m_R^\nu|$.[§] It is just assumption and we should explain why $|M_{\mathcal{N}}|$ is so huge comparing to other vector-like masses.

When $\mathcal{L}' \neq 0$, the neutrino mass matrix becomes

$$(\overline{\nu}_L, \overline{\nu}_R^c, \overline{N}_L, \overline{N}_R^c) \begin{pmatrix} 0 & 0 & \hat{y}^{\nu,L} v_L^* & y^{\nu,L} v_L^* \\ 0 & 0 & y^{\nu,R} v_R & \hat{y}^{\nu,R} v_R \\ \hat{y}^{\nu,LT} v_L^* & y^{\nu,RT} v_R & M_{NL} & M_N \\ y^{\nu,LT} v_L^* & \hat{y}^{\nu,RT} v_R & M_N^T & M_{NR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ N_L^c \\ N_R \end{pmatrix}, \quad (19)$$

where $\psi_L^c \equiv C\psi_L^T$ and $\hat{m}_{L,R}^\nu = \hat{y}^{\nu,L,R} v_{L,R}$. Under the condition, $(|m_L^\nu|, |\hat{m}_L^\nu| \ll |m_R^\nu|, |\hat{m}_R^\nu| \ll |M_{\mathcal{N}}|, |M_{NL}|, |M_{NR}|)$, the mass matrix for light six neutrinos is given by

$$M_{light} \simeq - \begin{pmatrix} \hat{m}_L^\nu & m_L^\nu \\ m_R^\nu & \hat{m}_R^\nu \end{pmatrix} \begin{pmatrix} M_{NL} & M_N \\ M_N^T & M_{NR} \end{pmatrix}^{-1} \begin{pmatrix} \hat{m}_L^{\nu T} & m_R^{\nu T} \\ m_L^{\nu T} & \hat{m}_R^{\nu T} \end{pmatrix}. \quad (20)$$

This mass matrix has been analyzed[3]. Here we show one example of the special limit, $\hat{m}_{L,R}^\nu = 0$, in which Eq.(20) becomes

$$M_{light} \sim \begin{pmatrix} -m_L^\nu M^{-1} m_L^{\nu T} & m_L^\nu M'^{-1} m_R^{\nu T} \\ m_R^\nu M'^{-1} m_L^{\nu T} & -m_R^\nu M''^{-1} m_R^{\nu T} \end{pmatrix}, \quad (21)$$

where M 's stands for functions of M_{NL}, M_{NR}, M_N .[¶] $|m_L^\nu| \ll |m_R^\nu|$ suggests that the order of neutrino mass matrix of the light three generation is given by

$$M_{light}^\nu \simeq -m_L^\nu M^{-1} m_L^{\nu T}, \quad (22)$$

which is the same as the usual seesaw mechanism. Notice that Eq.(22) holds independently of how larger m_R^ν is than m_L^ν as long as M 's are the same order. When M 's are of order 10^{14} GeV, the suitable tiny neutrino masses are obtained through the seesaw mechanism.

3 5D universal seesaw mechanism

Now we take the 5D setup, where the 5th dimension coordinate (y) is compactified on S^1/Z_2 . Only $SU(2)_L \times SU(2)_R$ singlets are spread in the 5D bulk, while the other quarks and leptons are localized on the 4D brane. We can consider two scenarios for the neutrino sector. One is the Dirac neutrino case and the other is Majorana neutrino case. We will show two cases in the following two subsections.

[§]Another possibility is that y^ν 's are extremely smaller than other Yukawa couplings of quarks and charged leptons.

[¶]We assume non-zero determinant of the inverse matrix of M_{NL}, M_{NR}, M_N in Eq.(20).

3.1 Dirac neutrino case

This case has the same Yukawa interactions for all quarks and leptons, which is corresponding to 4D case of $\mathcal{L}_0 = 0$ in Eq.(16). Quarks and leptons are all Dirac fields. Here we show the neutrino mass matrix, for an example. Other mass matrices of quarks and leptons are obtained in the same way.

Since the 5D theory is a vector-like theory, we must introduce the chiral partners N^m 's for N 's, which are mass dimension 3/2. Under Z_2 parity, $y \rightarrow -y$, $N_L \equiv (N_l, N_l^m)^T$ ($N_R \equiv (N_r^m, N_r)^T$) transforms as $N_L(x^\mu, -y) = \gamma^5 N_L(x^\mu, y)$ ($N_R(x^\mu, -y) = -\gamma^5 N_R(x^\mu, y)$). The mode expansions are given by

$$N_L(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \left(\begin{array}{c} \frac{1}{\sqrt{2}} N_l^{(0)}(x^\mu) + \sum_{n=1}^{\infty} \cos(\frac{ny}{R}) N_l^{(n)}(x^\mu) \\ \sum_{n=1}^{\infty} \sin(\frac{ny}{R}) N_l^{m(n)}(x^\mu) \end{array} \right), \quad (23)$$

$$N_R(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \left(\begin{array}{c} \sum_{n=1}^{\infty} \sin(\frac{ny}{R}) N_r^{m(n)}(x^\mu) \\ \frac{1}{\sqrt{2}} N_r^{(0)}(x^\mu) + \sum_{n=1}^{\infty} \cos(\frac{ny}{R}) N_r^{(n)}(x^\mu) \end{array} \right), \quad (24)$$

where the factor $1/\sqrt{2}$ of the zero-mode is needed for the canonical kinetic term in the 4D effective Lagrangian. N_l, N_r^m (N_r, N_l^m) are left- (right-)handed Weyl fermions, in which N_l (N_r) has a zero-mode and survive below the compactification scale, R^{-1} .

The bulk Dirac mass of the vector-like neutrinos is given by

$$\mathcal{L}_{5\text{DM}} = M_{\mathcal{N}ij} \overline{N_{Li}} N_{Rj} + \text{h.c.}, \quad (25)$$

which is invariant under the Z_2 parity, and conserves the lepton number. We can always take the diagonal base of the generation index for $M_{\mathcal{N}ij}$. We assume their eigenvalues are real for simplicity in the following discussions. By integrating out the 5th dimension, the 4D effective Lagrangian is obtained, where Eq.(25) becomes

$$\mathcal{L}_{4\text{DM}} = \sum_{n=0}^{\infty} M_{\mathcal{N}} \overline{N_l^{(n)}} N_r^{(n)} + \sum_{n=1}^{\infty} M_{\mathcal{N}} \overline{N_l^{m(n)}} N_r^{m(n)} + \text{h.c.}. \quad (26)$$

We omit spinor indices here. Kaluza-Klein (KK) masses are given by

$$\mathcal{L}_{\text{KK}} = \sum_{n=1}^{\infty} \frac{n}{R} (\overline{N_{l,r}^{m(n)}} N_{l,r}^{(n)} + \overline{N_{l,r}^{(n)}} N_{l,r}^{m(n)}). \quad (27)$$

The Dirac mass terms between the bulk fields N 's and the brane-localized lepton doublets L 's are given by

$$\mathcal{L}_{4\text{Dm}} = \frac{1}{\sqrt{M_*}} \sum_{n=0}^{\infty} (y_{ij}^{\nu, L} \overline{L_{Li}} \tilde{H}_L N_{rj}^{(n)} + y_{ij}^{\nu, R} \overline{L_{Ri}} \tilde{H}_R N_{lj}^{(n)}) \delta(y) + \text{h.c.}, \quad (28)$$

where M_* is the 5D Planck scale. Then the 4D neutrino mass matrix is given by

$$\mathcal{L}_{m_\nu} = \left(\begin{array}{c|ccc} \overline{\nu}_L, & \overline{N}_r^{m(1)}, & \overline{N}_r^{m(2)}, & \dots & \overline{N}_l^{(0)}, & \overline{N}_l^{(1)}, & \overline{N}_l^{(2)}, & \dots \end{array} \right) \times \left(\begin{array}{c|ccc} 0 & 0 & 0 & \dots & \frac{m_L^\nu}{\sqrt{2\pi R M_*}} & \frac{m_L^\nu}{\sqrt{\pi R M_*}} & \frac{m_L^\nu}{\sqrt{\pi R M_*}} & \dots \\ & M_{\mathcal{N}} & & & & \frac{1}{R} & \frac{2}{R} & \\ & & M_{\mathcal{N}} & & & & & \\ \vdots & & & \ddots & M_{\mathcal{N}} & & & \ddots \\ \hline \frac{m_R^{\nu T}}{\sqrt{2\pi R M_*}} & & & & & & & \\ \frac{m_R^{\nu T}}{\sqrt{\pi R M_*}} & & \frac{1}{R} & & & M_{\mathcal{N}} & & \\ \frac{m_R^{\nu T}}{\sqrt{\pi R M_*}} & & & \frac{2}{R} & & & M_{\mathcal{N}} & \\ \vdots & & & & & & & \ddots \end{array} \right) \left(\begin{array}{c} \nu_R \\ N_l^{m(1)} \\ N_l^{m(2)} \\ \vdots \\ N_r^{(0)} \\ N_r^{(1)} \\ N_r^{(2)} \\ \vdots \end{array} \right), \quad (29)$$

where $m_{L,R}^\nu$ is triangular matrix in the flavor space as Eq.(9).

Let us pick up the sub-matrix of n -mode from Eq.(29). Since the KK mass, n/R , is proportional to the unit matrix in the flavor space, all n -mode fields are diagonalized simultaneously in the flavor space as

$$\left(\begin{array}{cc} M_{\mathcal{N}} & \frac{n}{R} \\ \frac{n}{R} & M_{\mathcal{N}} \end{array} \right), \quad (30)$$

where $M_{\mathcal{N}} = \text{diag}(M_{\nu e}, M_{\nu \mu}, M_{\nu \tau})$. Since the inverse mass matrix of Eq.(30) is given by

$$\frac{1}{M_{\mathcal{N}k}^2 - (\frac{n}{R})^2} \left(\begin{array}{cc} M_{\mathcal{N}k} & -\frac{n}{R} \\ -\frac{n}{R} & M_{\mathcal{N}k} \end{array} \right) \quad (31)$$

(k : generation index) the summation of the infinite numbers of “seesaw” is calculated as

$$\begin{aligned} m_{ij}^\nu &= -\frac{(m_L^\nu)_{ik}}{\pi R M_*} \left[\frac{1}{2M_{\mathcal{N}k}} + \sum_{n=1}^{\infty} \frac{M_{\mathcal{N}k}}{M_{\mathcal{N}k}^2 - (\frac{n}{R})^2} \right] (m_R^{\nu T})_{kj} \\ &= -(m_L^\nu)_{ik} \frac{1}{2M_* \tan[\pi R M_{\mathcal{N}k}]} (m_R^{\nu T})_{kj}, \end{aligned} \quad (32)$$

when the magnitude of $\min[M_{\mathcal{N}k} \pm \frac{n}{R}]$ is much larger than the Dirac mass scale^{||}. This means that the 5D universal seesaw formula is obtained by the replacement

$$M_k \rightarrow 2M_* \tan[\pi R M_k] \quad (33)$$

from the 4D formula[10][12]. Other quarks and leptons' mass matrices are obtained in the same way. We should notice that the condition $|m_L| (= \mathcal{O}(10^2) \text{ GeV}) \ll |m_R| \ll$

^{||}The case of $|M_k| \ll R^{-1}$ reproduces the ordinal 4D universal seesaw formula with the volume suppression factor, $(2\pi R M_*)^{-1}$.

$|M_k|, R^{-1}$ is needed for the universal seesaw formula and $|m_R/(2M_* \tan[\pi R M_k])|$ should be of $\mathcal{O}(10^{-2})$ for the suitable mass scale of the bottom quark and tau lepton. They suggest

$$M_* \sim R^{-1} = \mathcal{O}(10^{18}) \text{ GeV}, \quad |m_R^{b,\tau}/M_k| = \mathcal{O}(10^{-2}). \quad (34)$$

Other quarks and charged lepton masses can be reproduced from the hierarchies of $m_{L,R}$. We assume T, \bar{T} are not bulk fields and $|m_R^t|$ should be the same order as (or even larger than) $|M_T|$ in order to reproduce the heavy top quark mass as the 4D case.

The tangent function is a feature in the 5D seesaw, which has the significant effects when $|M_k| \sim R^{-1}$. Especially, when $|M_k| = \frac{1}{2R}$, the light degrees of freedom become massless by the infinite times “seesaw”, $-\frac{1}{2\pi R M_*} \times ([\frac{1}{2R}]^{-1} - [\frac{1}{2R}]^{-1} + [\frac{3}{2R}]^{-1} - [\frac{3}{2R}]^{-1} + \dots) \rightarrow 0$. Therefore, we can explain the smallness of Dirac neutrino masses by taking bulk neutrino mass $|M_{\mathcal{N}k}|$ close to $\frac{1}{2R}$. In 4D, the smallness of neutrino masses needed an unnatural assumption of extremely huge $|M_{\mathcal{N}}|$. In 5D, tiny neutrino masses can be naturally realized thanks to the tangent function. The neutrino masses can be tiny even when $|M_{\mathcal{N}}|$ is the same order as other bulk masses. The condition of $|M_{\mathcal{N}k}|$ being very close to $\frac{1}{2R}$ makes the neutrino masses tiny comparing to other quarks and charged leptons’ masses.

3.2 Majorana neutrino case

Next, let us show the Majorana neutrino case which corresponds to the 4D case of $\mathcal{L}' \neq 0$ in Eq.(16). The quarks and charged lepton sectors are the completely same as the previous subsection. The difference exists only in the neutrino sector. We take the bulk neutrino masses as

$$\mathcal{L}_{5\text{DM}} = M_{NLij} \overline{N_{Li}^c} N_{Lj} + M_{NRij} \overline{N_{Ri}^c} N_{Rj} + M_{\mathcal{N}ij} \overline{N_{Li}} N_{Rj} + \text{h.c.} \quad (35)$$

These masses are invariant under the Z_2 parity. The 1st and 2nd masses break the lepton number. By integrating out the 5th dimensional space, the 4D effective Lagrangian is obtained, in which Eq.(35) becomes

$$\begin{aligned} \mathcal{L}_{4\text{DM}} &= \sum_{n=0}^{\infty} (M_{NL} \overline{N_l^c}^{(n)} N_l^{(n)} + M_{NR} \overline{N_r^c}^{(n)} N_r^{(n)} + M_{\mathcal{N}} \overline{N_l}^{(n)} N_r^{(n)}) + \text{h.c.} \\ &- \sum_{n=1}^{\infty} (M_{NL} \overline{N_l^{mc}}^{(n)} N_l^{m(n)} + M_{NR} \overline{N_r^{mc}}^{(n)} N_r^{m(n)} - M_{\mathcal{N}} \overline{N_l^m}^{(n)} N_r^{m(n)}) + \text{h.c.} \end{aligned} \quad (36)$$

KK masses are given by Eq.(27). The Dirac mass terms between the bulk fields N 's and the brane-localized lepton doublets L 's are given by

$$\begin{aligned} \mathcal{L}_{4\text{Dm}} &= \frac{1}{\sqrt{M_*}} \sum_{n=0}^{\infty} (y_{ij}^{u,L} \overline{L_{Li}} \tilde{H}_L N_{rj}^{(n)} + y_{ij}^{u,R} \overline{L_{Ri}} \tilde{H}_R N_{lj}^{(n)} \\ &+ \hat{y}_{ij}^{u,L} \overline{N_{Li}} \tilde{H}_L N_{lj}^{c(n)} + \hat{y}_{ij}^{u,R} \overline{N_{Ri}} \tilde{H}_R N_{rj}^{c(n)}) \delta(y) + \text{h.c.} \end{aligned} \quad (37)$$

Then the 4D neutrino mass matrix is given by

$$\begin{pmatrix} \overline{\nu}_L, & \overline{\nu}_R^c, & \overline{N}_l^{(0)}, & \overline{N}_r^{c(0)}, & \overline{N}_l^{(n)}, & \overline{N}_l^{mc(n)}, & \overline{N}_r^{c(n)}, & \overline{N}_r^{m(n)} \end{pmatrix} \times \\
 \begin{pmatrix} 0 & 0 & \frac{\hat{m}_L^\nu}{\sqrt{2\pi RM_*}} & \frac{m_L^\nu}{\sqrt{2\pi RM_*}} & \frac{\hat{m}_L^\nu}{\sqrt{\pi RM_*}} & 0 & \frac{m_L^\nu}{\sqrt{\pi RM_*}} & 0 \\ 0 & 0 & \frac{m_R^\nu}{\sqrt{2\pi RM_*}} & \frac{\hat{m}_R^\nu}{\sqrt{2\pi RM_*}} & \frac{m_R^\nu}{\sqrt{\pi RM_*}} & 0 & \frac{\hat{m}_R^\nu}{\sqrt{\pi RM_*}} & 0 \\ \frac{\hat{m}_L^{\nu T}}{\sqrt{2\pi RM_*}} & \frac{m_R^{\nu T}}{\sqrt{2\pi RM_*}} & M_{NL} & M_N & 0 & 0 & 0 & 0 \\ \frac{m_L^{\nu T}}{\sqrt{2\pi RM_*}} & \frac{\hat{m}_R^{\nu T}}{\sqrt{2\pi RM_*}} & M_N^T & M_{NR} & 0 & 0 & 0 & 0 \\ \frac{\hat{m}_L^{\nu T}}{\sqrt{\pi RM_*}} & \frac{m_R^{\nu T}}{\sqrt{\pi RM_*}} & 0 & 0 & M_{NL} & \frac{n}{R} & M_N & 0 \\ 0 & 0 & 0 & 0 & \frac{n}{R} & -M_{NL} & 0 & M_N \\ \frac{m_L^{\nu T}}{\sqrt{\pi RM_*}} & \frac{\hat{m}_R^{\nu T}}{\sqrt{\pi RM_*}} & 0 & 0 & M_N^T & 0 & M_{NR} & \frac{n}{R} \\ 0 & 0 & 0 & 0 & 0 & M_N^T & \frac{n}{R} & -M_{NR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ N_l^{c(0)} \\ N_r^{(0)} \\ N_l^{c(n)} \\ N_l^{m(n)} \\ N_r^{(n)} \\ N_r^{mc(n)} \end{pmatrix} \quad (38)$$

where $n = 1 \sim \infty$. This is an infinite times infinite matrix. In the 4D limit, $|M_{NL}|, |M_{NR}|, |M_N| \ll R^{-1}$, the infinite mass matrix Eq.(38) reduces to 4D mass matrix Eq.(19).

Here let us consider some special limits. The first is $M_{NL,R} = 0$ and $\hat{m}_{L,R}^\nu = 0$ which is the limit of previous subsection, and can not induce seesaw mechanism because it does not break lepton number. Next is the limit of $M_N = 0$ and $\hat{m}_{L,R}^\nu = 0$. This case makes Eq.(38) become

$$\begin{pmatrix} \overline{\nu}_L, & \overline{N}_r^{m(n)} | & \overline{N}_r^{c(0)}, & \overline{N}_r^{c(n)} \parallel & \overline{\nu}_R^c, & \overline{N}_l^{mc(n)} | & \overline{N}_l^{(0)}, & \overline{N}_l^{(n)} \end{pmatrix} \times \\
 \begin{pmatrix} 0 & 0 & \frac{m_L^\nu}{\sqrt{2\pi RM_*}} & \frac{m_L^\nu}{\sqrt{\pi RM_*}} \parallel & 0 & 0 & 0 & 0 \\ 0 & -M_{NR} & 0 & \frac{n}{R} \parallel & 0 & 0 & 0 & 0 \\ \hline \frac{m_L^{\nu T}}{\sqrt{2\pi RM_*}} & 0 & M_{NR} & 0 & 0 & 0 & 0 & 0 \\ \frac{m_L^{\nu T}}{\sqrt{\pi RM_*}} & \frac{n}{R} & 0 & M_{NR} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & \frac{m_R^\nu}{\sqrt{2\pi RM_*}} & \frac{m_R^\nu}{\sqrt{\pi RM_*}} \\ 0 & 0 & 0 & 0 & 0 & -M_{NL} & 0 & \frac{n}{R} \\ \hline 0 & 0 & 0 & 0 & \frac{m_R^{\nu T}}{\sqrt{2\pi RM_*}} & 0 & M_{NL} & 0 \\ 0 & 0 & 0 & 0 & \frac{m_R^{\nu T}}{\sqrt{\pi RM_*}} & \frac{n}{R} & 0 & M_{NL} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_r^{mc(n)} \\ - \\ N_r^{(0)} \\ N_r^{(n)} \\ \overline{\nu}_R \\ N_l^{m(n)} \\ - \\ N_l^{c(0)} \\ N_l^{c(n)} \end{pmatrix}.$$

Two same structures appear in upper-left and downer-right parts in this matrix. This means two 3×3 mass matrices are obtained through the seesaw mechanism, when $|m_{L,R}^\nu| \ll |M_{NL,R}|, R^{-1}$. They are given by [10][13]-[15]

$$m_{ij}^{\nu_l} \simeq -(m_L^\nu)_{ik} \frac{1}{2M_* \tanh[\pi RM_{NRk}]} (m_L^{\nu T})_{kj}, \quad (39)$$

$$m_{ij}^{\nu_r} \simeq -(m_R^\nu)_{ik} \frac{1}{2M_* \tanh[\pi RM_{NLk}]} (m_R^{\nu T})_{kj}. \quad (40)$$

When $|m_R^\nu/M_k| = \mathcal{O}(10^{-2})$ is satisfied as Eq.(34), the bulk mass $|M_{NRk}|$ should be of $\mathcal{O}(10^{14})$ GeV for the suitable mass scale $|m^\nu| = \mathcal{O}(10^{-1})$ eV of the three generation

active neutrinos. It is natural to consider $|M_{\mathcal{N}Rk}| \sim |M_{\mathcal{N}Lk}|$, which suggests $|m^{\nu_r}| = \mathcal{O}(10^{10})$ GeV from Eq.(34). The limit, $M_{\mathcal{N}} = 0$ and $m_{L,R} = 0$, changes Eqs.(39) and (40) as

$$m_{ij}^{\nu_l} \simeq -(m_L^\nu)_{ik} \frac{1}{2M_* \tanh[\pi R M_{\mathcal{N}Lk}]} (m_L^{\nu T})_{kj}, \quad (41)$$

$$m_{ij}^{\nu_r} \simeq -(m_R^\nu)_{ik} \frac{1}{2M_* \tanh[\pi R M_{\mathcal{N}Rk}]} (m_R^{\nu T})_{kj}. \quad (42)$$

Both limits satisfy the 4D relation

$$|m^{\nu_l}| |m^{\nu_r}| \sim |m^\tau|^2 \quad (43)$$

when $|M_{\mathcal{N}L,R}| \sim |M_{\mathcal{E}}|$.

4 Summary

We have shown the universal seesaw mechanism in the flat extra dimension setup. The 5D universal seesaw has been demonstrated in the framework of the left-right symmetric model. We introduce extra vector-like isosinglet fields in the 5D bulk. The universal seesaw formula is easily obtained as a replacement of the vector-like mass in 4D case M_i to $2M_* \tanh[\pi R M_i]$ (M_* : 5D Planck scale, M_i : vector-like bulk mass, and R : compactification radius). As for the neutrinos, the smallness of Dirac neutrino masses can be naturally explained in the 5D setup. Only Dirac neutrino masses can be very small when the bulk neutrino masses are close to a half of compactification scale. We have also shown the Majorana neutrino case, where 5D seesaw mechanism works. For the large top mass, the vector-like top quarks should be localized on the 4D brane.

We can also consider the universal seesaw mechanism in the frame work of $SU(5)$ GUT. When heavy three generation vector-like **10** and **1** fields and singlet Higgs field are introduced in the bulk, the universal seesaw mechanism works in the 5D $SU(5)$ GUT[8]. The 5D universal seesaw calculations are easily achieved by the replacement of $\mu_{10} \rightarrow \tanh[2\pi R \mu_{10}]$ in Eqs.(3.6)-(3.8) and $\langle 1'_H \rangle \rightarrow \tanh[2\pi R \langle 1'_H \rangle]$ in Eq.(3.9) of Ref.[8].

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